College Algebra Honors
Midterm Review

Name KEY
Date

ALGEBRAIC EXPRESSIONS:

1. Identify the property illustrated by the following: $-5 + [\,0 + (-2)] = -5 + (-2)$

Identity Property of Addition

2. Simplify: $-3^2(2) - 12 + 2^3$

$-9(2)-12+8$

$-18-12+8 = -30+8 = -22$

3. List the elements of the set $\{-\sqrt{11}, -7, -\frac{4}{5}, \sqrt{9}, \frac{12}{2}\}$ that are whole numbers.

$\boxed{\sqrt{9}, \frac{12}{2}}$

4. When written without absolute value bars $|20 - x^2|$ if $-4 \le x \le 4$ is equal to:

$\boxed{20 - x^2}$

POLYNOMIALS:

1. $(8a^3 + 9a^2 - 5p) + (4a^3 + 12a^2 + 9)$ equals: $\boxed{4a^3 + 21a^2 - 5p - 9}$

2. The coefficient of x^2 in the product $(4x^2 - 2x)(-5x^2 + 7x + 1)$ is: $4x^2 - 14x^2 = \boxed{-10x^2}$

3. Expand: $(5x^a + 1)^2$ $\boxed{25x^{2a} + 10x^a + 1}$

4. Find the second term of the quotient when $20x^3y^4 + 3x^3y^2 - xy^2 - 14y - 12$ is divided by $2y + 5$

$-25x^3y^2$

$$\begin{array}{r} 10x^3y^3 - 25x^3y^2 \\ 2y+5\,)\overline{20x^3y^4 + 3x^3y^2} \\ -20x^3y^4 + 50x^3y^3 \\ \hline -50x^3y^3 \end{array}$$

5. Add: $\frac{2ax}{y} + \frac{r}{3s} = \frac{6asx}{3sy} + \frac{ry}{3sy} = \boxed{\frac{6asx + ry}{3sy}}$

6. Multiply: $\frac{a+b}{2a^2 - 2ab} \cdot \frac{4a^2 - 6ab + 2b^2}{a^2 + 2ab + b^2} = \frac{\cancel{a+b}}{\cancel{2}a(\cancel{a-b})} \cdot \frac{\cancel{2}(2a-b)(\cancel{a-b})}{(a+b)\cancel{(a+b)}} = \boxed{\frac{2a-b}{a(a+b)}}$

$2(2a^2 - 3ab + b^2)$

7. The expression: $\dfrac{\frac{x+1}{x^2 - x - 6}}{\frac{3}{x-3}}$ equals: $\frac{x+1}{\cancel{(x-3)}(x+2)} \cdot \frac{\cancel{x-3}}{3} = \frac{x+1}{3(x+2)}$

8. Factor: $x^2 - y^2 + x^2 - 2xy + y^2$ → OR $2x^2 - 2xy$ $\boxed{2x(x-y)}$

$(x-y)(x+y) + (x-y)(x-y)$

$(x-y)[(x+y) + (x-y)] = \boxed{(x-y)(2x)}$

9. Factor: $64a^3 + 27b^6$? $\boxed{(4a+3b^2)(16a^2-12ab+9b^4)}$

10. Expand: $(x - 3y - z)^2$

$(x-3y-z)(x-3y-z)$

$x^2-3xy-xz$
$-3xy+9y^2+3yz$
$-xz+3yz+z^2$

$\boxed{x^2-6xy-2xz+9y^2+6yz+z^2}$

EXPONENTS AND RADICALS:

1. Simplify: $\dfrac{8a^{-\frac{1}{5}}b^{\frac{3}{4}}c^{-6}}{2b^{\frac{1}{3}}c^{-2}} = \dfrac{8b^{3/4}c^2}{2a^{1/5}b^{1/3}c^6} = \boxed{\dfrac{4b^{5/12}}{a^{1/5}c^4}}$

$\frac{3}{4}-\frac{1}{3}=\frac{9}{12}-\frac{4}{12}=\frac{5}{12}$

2. Evaluate: $\left(\dfrac{16}{81}\right)^{\frac{3}{4}} = \sqrt[4]{\dfrac{(2^4)^3}{(3^4)^3}} = \sqrt[4]{\dfrac{2^{12}}{3^{12}}} = \dfrac{2^3}{3^3} = \boxed{\dfrac{8}{27}}$ OR $\left(\dfrac{\sqrt{16}}{\sqrt{81}}\right)^3 = \left(\dfrac{2}{3}\right)^3 = \boxed{\dfrac{8}{27}}$

3. Reduce: $\sqrt[6]{27} = \sqrt[6]{3^3} = 3^{3/6} = 3^{1/2} = \boxed{\sqrt{3}}$

4. Subtract: $5\sqrt{27} - 2\sqrt{12} = 5\sqrt{9\cdot 3} - 2\sqrt{4\cdot 3} = 15\sqrt{3} - 4\sqrt{3} = \boxed{11\sqrt{3}}$

5. Multiply: $(5+\sqrt[3]{3})(7-\sqrt[3]{3}) = 35 - 5\sqrt[3]{3} + 7\sqrt[3]{3} - \sqrt[3]{9} = \boxed{35 - 2\sqrt[3]{3} - \sqrt[3]{9}}$

6. Simplify: $\dfrac{5}{\sqrt{2}-\sqrt{7}} \cdot \dfrac{\sqrt{2}+\sqrt{7}}{\sqrt{2}+\sqrt{7}} = \dfrac{5\sqrt{2}+5\sqrt{7}}{2-7} = \dfrac{5\sqrt{2}+5\sqrt{7}}{-5} = \boxed{-\sqrt{2}-\sqrt{7}}$

7. Rationalize: $\sqrt[4]{\dfrac{2}{27}} = \dfrac{\sqrt[4]{2}}{\sqrt[4]{3^3}} \cdot \dfrac{\sqrt[4]{3}}{\sqrt[4]{3}} = \dfrac{\sqrt[4]{6}}{\sqrt[4]{3^4}} = \boxed{\dfrac{\sqrt[4]{6}}{3}}$

COMPLEX NUMBERS:

1. $(7 - 8i) - (1 - 13i)$ equals: $\boxed{6+5i}$

2. Multiply: $\sqrt{-5}\cdot\sqrt{-12}$ $\quad i\sqrt{5} \cdot 2i\sqrt{3} = 2i^2\sqrt{15} = \boxed{-2\sqrt{15}}$

3. $(3 + 2i)(3 - 7i)$ equals: $9 - 21i + 6i - 14i^2 = \boxed{23 - 15i}$

4. The conjugate of 5 - i is: $\boxed{5+i}$

9. The discriminant for the equation $2x^2 + 5x = -4$ is: $2x^2+5x+4=0$

$a=2$, $b=5$, $c=4$ $\quad b^2-4ac$

$25-4(2)(4) = 25-32 = \boxed{-7}$

10. Solve $\sqrt{2x}-1=\sqrt{2x-7}$

$2x - 2\sqrt{2x} + 1 = 2x-7 \qquad \sqrt{2x} = 4 \qquad \boxed{x=8}$

$-2\sqrt{2x} = -8 \qquad 2x = 16$

11. Use the process of completing the square to solve the equation $3x^2 + 12x = 9$.

$x^2+4x=3$

$x^2+4x+4=7$

$(x+2)^2=7$

$x+2=\pm\sqrt{7}$

$\boxed{x=-2\pm\sqrt{7}}$

12. Solve: $x^3 + 3x^2 - 4x - 12 = 0$

$x^2(x+3)-4(x+3)=0$

$(x+3)(x^2-4)=0$

$(x+3)(x-2)(x+2)=0$

$x=-3 \quad x=\pm 2$

$\boxed{\{-3,-2,2\}}$

13. Find the real solutions for: $\sqrt[3]{11x+4}=-5$

$11x+4=-125 \qquad 11x=-129 \qquad \boxed{x=\frac{-129}{11}}$

14. Jon can take 50 orders in 3 hours, and Josh can take 50 ordes in 5 hours. How long will it take both Jon and Josh, working together, to take 100 orders?

See below

skip 15. Suppose y varies directly as t and inversely as z. If y = 8 when t = 12 and z = 6, find y when t = 20 and z = 4.

$y=\frac{kt}{z}$, $8=\frac{k(12)}{6}$ $\quad 48=k(12)$ $\quad k=4$ $\quad y=\frac{4t}{z}$ $\quad y=\frac{4(20)}{4}$ $\quad \boxed{y=20}$

skip 16. a varies jointly with c and d and inversely with the square of p. What is the result on a if c is halved, d is tripled, and p is doubled?

$a=\frac{kcd}{p^2}$ $\qquad a=\frac{k(\frac{c}{2})3d}{(2p)^2}$ $\qquad a=\frac{3kcd}{8p^2}$ $\qquad$ a is multiplied by $\frac{3}{8}$

Josh would take 10 hrs to do 100 orders

Jon would take 6 hrs to do 100 orders

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	rate/hr	(hours) time	Part of work done
Jon	$\frac{1}{10}$	t	$\frac{t}{10}$
Josh	$\frac{1}{6}$	t	$\frac{t}{6}$

let t = time, working together, it takes for J+J to complete 100 orders

Jon's part + Josh's Part = Whole order

$$\left(\frac{t}{10}+\frac{t}{6}=1\right)60$$

$$6t+10t=60$$

$$16t=60$$

$$t=\frac{60}{16}=\frac{15}{4}=3\frac{3}{4}$$

It would take $3\frac{3}{4}$ hours for Jon & Josh to complete 100 orders.

5. i^{351} in simpliest form is: $i^{3} = \boxed{-i}$

BINOMIAL THEOREM/COUNTING THEORY:

1. $28a^6b^2$ is a term of a binomial expansion of $(a + b)^8$, which one? $\boxed{\text{third}}$

2. Which row of Pascal's triangle could be used to expand $(3x^2 - 7y^3)^5$? $\boxed{\text{Fifth}}$ $\boxed{1\ 5\ 10\ 10\ 5\ 1}$

3. Find the value of P(10,5). $= \dfrac{10!}{(10-5)!} = \dfrac{10!}{5!} = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = \boxed{30240}$

4. There are 13 girls in a third grade class. In how many ways can the teacher select a group of 4 girls to participate in a special project?

$${}_{13}C_4 = \frac{13!}{4!(13-4)!} = \frac{13!}{4!\,9!} = \frac{13 \cdot 12 \cdot 11 \cdot 10}{4 \cdot 3 \cdot 2 \cdot 1} = \boxed{715 \text{ ways}}$$

SOLVING EQUATIONS:

1. Solve $3(1 - 2z) - 3 = -9(1 + 2z) + 3z$ for z. $\boxed{z = -1}$

$3 - 6z - 3 = -9 - 18z + 3z$

$9z = -9$

$z = -1$

2. The solution to the equation $\left(\dfrac{5}{x+3} - \dfrac{1}{x-2} = \dfrac{-6}{x+3}\right)(x+3)(x-2)$ is:

$x \neq 2$
$x \neq -3$

$5(x-2) - 1(x+3) = -6(x-2)$

$5x - 10 - x - 3 = -6x + 12 \rightarrow 10x = 25$

$\boxed{x = 5/2}$

3. Solve $a = \dfrac{3a}{1-r} - 5$ for r.

$a + 5 = \dfrac{3a}{1-r} \rightarrow 1 - r = \dfrac{3a}{a+5} \qquad -r = \dfrac{3a}{a+5} - 1$

$r = 1 - \dfrac{3a}{a+5} \rightarrow r = \dfrac{a+5}{a+5} - \dfrac{3a}{a+5} \rightarrow \boxed{r = \dfrac{5-2a}{a+5} \quad a \neq -5}$

4. The perimeter of a rectangle is 38 meters. The length is 1 meter less than 3 times the width. Find the width of the rectangle.

3w-1

w

$P = 2l + 2w$

$38 = 2(3w-1) + 2w$

$19 = 3w - 1 + w$

$20 = 4w$

$5 = w$

$l = 5 \cdot 3 - 1 = 14$

$\boxed{\text{width} = 5\text{ m},\ \text{length} = 14\text{ m}}$

5. Two trains leave the same point at the same time traveling in the opposite directions. One travels 5 mph faster than the other. After 5 hours they are 375 miles apart. What is the speed of the faster train?

let r = rate of train 1

train?	r	t	d
1	r	5	5r
2	r+5	5	5(r+5)

$5r + 5(r+5) = 375$

$5r + 5r + 25 = 375$

$10r = 350$

$r = 35$ mph

$\boxed{40 \text{ mph}}$

6. Solve the equation $|5a - 2| = |4a + 8|$.

skip

$5a - 2 = 4a + 8$ or $5a - 2 = -4a - 8$

$a = 10$ or $9a = -6$

$a = -\frac{2}{3}$

$\boxed{\left\{10, -\frac{2}{3}\right\}}$

**7. Solve $2x(x - 5) = 2x - 9$ for x.

$2x^2 - 10x = 2x - 9$

$2x^2 - 12x + 9 = 0$

$a = 2$
$b = -12$
$c = 9$

$x = \dfrac{12 \pm \sqrt{144 - 4(2)(9)}}{4} = \dfrac{12 \pm \sqrt{72}}{4} = \dfrac{12 \pm 6\sqrt{2}}{4} = \dfrac{6 \pm 3\sqrt{2}}{2}$

8. What kind of solutions does $(4x - 2)^2 = 9$ have?

$16x^2 - 16x + 4 = 9$

$16x^2 - 16x - 5 = 0$

$a = 16 \quad b = -16 \quad c = -5$

$b^2 - 4ac$

$(-16)^2 - 4(16)(-5)$

$256 + 320 = 576$

2 Real Solutions
Rational